8.4 Q^n 5

Solution

a) Any ~~skec~~ sketch of a sine curve with an initial crest at (1, 900) and a final trough at (20, 300) will be good for the point.

b) if we divide the 20 mile stretch into slices then

$20 = N \Delta x$ and $\Delta x = 20/N$.

approximating the density of the number of cars with $f(x)$ the Riemann sum that approximates the total number of cars is

$$\sum_{i=1}^{N} \delta(x_i)\, \Delta x = \sum_{i=1}^{N} \left[300\left(2 + \sin\left(4\sqrt{x_i + 0.15}\right)\right)\right] \Delta x.$$

c) the total number of cars on the stretch is

$$C = \int_0^{20} \left(600 + 300 \sin 4\sqrt{x + 0.15}\right) dx \quad \text{as } N \to \infty$$

$$= \int_0^{20} 600\, dx + 300 \int_0^{20} \left(\sin 4\sqrt{x + 0.15}\right) dx$$

$$\int_0^{20} \left(\sin 4\sqrt{x+0.15}\right) dx$$

let $\sqrt{x+0.15} = w$

then $x = w^2 - 0.15$

and $dx = 2w\,dx$

$$\int_{w=\sqrt{0.15}}^{w=\sqrt{20.15}} \sin 4w \cdot 2w\,dw$$

$$= 2\int_{0.15}^{20.15} w \sin 4w\,dw$$

$$= 2\left[-\tfrac{1}{4}w\cos 4w + \tfrac{1}{16}\sin 4w\right]_{0.15}^{20.15}$$

$$\simeq -1.624$$

also $\int_0^{20} 600\,dx = 600x \Big|_0^{20}$

$$= 1200$$

$\therefore\ C = 1200 + 300(-1.624)$

$= 1151.3$

Comments

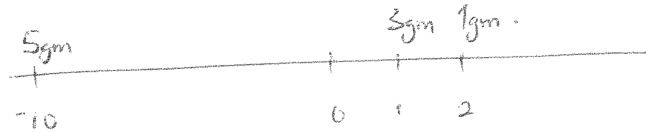
This was a long quiz and an ~~any~~ attempt close to the condition in (a) merited the point.

Most of you did well on (b) except a few who did not recognise that we can estimate the number of cars with the function $f(x)$ and small changes in $x = 20$ (Δx).

The problem with (c) was that most of you did not know how to integrate the integrand or ran out time.

8.4 Qu8

Solution



if $\bar{x}$ is the centre of mass of the system

Given

masses $m_1 = 5gm$, $m_2 = 3gm$ and $m_3 = 1gm$

located at $x_1 = -10$, $x_2 = 1$ and $x_3 = 2$ respectively

from the formula for centre of mass:

$$\bar{x} = \frac{\sum x_i m_i}{\sum m_i} = \frac{(-10)5 + 1(3) + 2(1)}{5 + 3 + 1}$$

$$= -45/9$$

$$= -5$$

Comments.

Most of you answered this question quiet well. just a few of you could not ~~use~~ relate the information given to the formula.

8·4

Q^{u}21.

a) We expect the centre of mass of the rod to be to the ~~rod~~ right of the origin as density is minimum at $x = -1$ and increases as x increases.

Hence more of the rod's mass is on the right half of the rod.

b)

$$x\rho = \int_{-1}^{1} x(3 - e^{-x})\,dx$$

$$= \frac{3x^2}{2} + xe^{-x} + e^{-x} \Big|_{-1}^{1} \quad \text{by integration by parts}$$

$$= \left(\tfrac{3}{2} + e^{-1} + e^{-1}\right) - \left(\tfrac{3}{2} - e^{1} + e^{1}\right)$$

$$= 2/e.$$

total mass

$$\int_{-1}^{1} (3 - e^{-x})\, dx = 3x + e^{-x} \Big|_{-1}^{1} = 6 - e + 1/e$$

Coordinate of centre of mass

$$\bar{x} = \frac{2/e}{6 - e + 1/e} = \frac{2}{1 + 6e - e^2} \approx 0.2$$

Comments:

part (a) of the quiz was answered well by most students

for part (b) quite a number of you did not realize that the centre of mass can be obtained from

$$\bar{x} = \frac{\int x \delta}{\int \delta}$$

and the few who did, could not integrate the function $x(3 - e^{-x})$ well.